

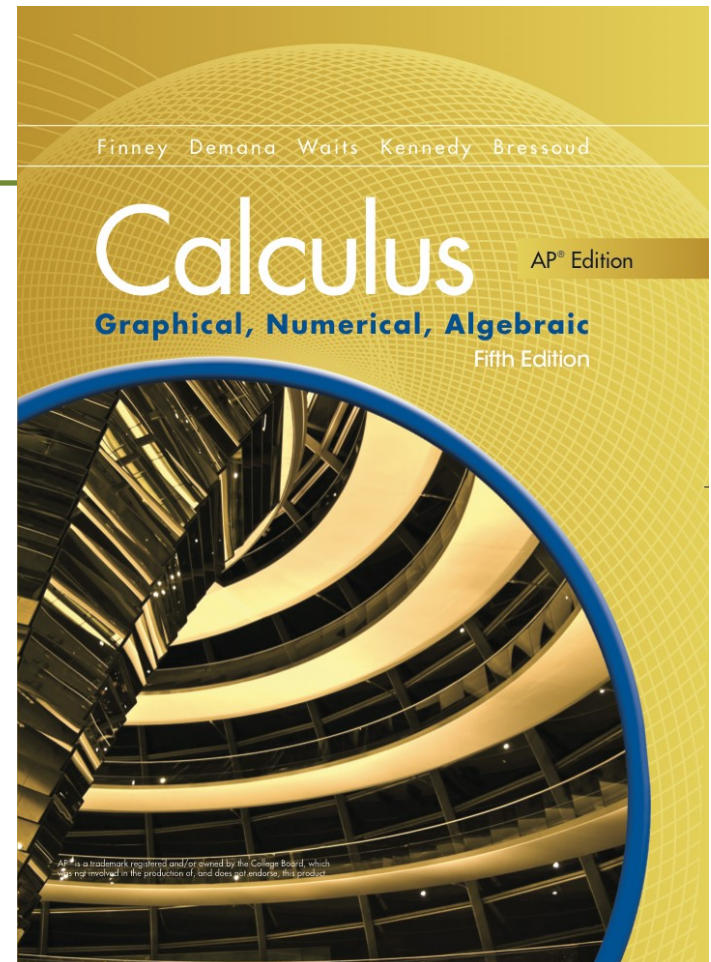
Chapter 2

Limits and Continuity



Section 2.3

Continuity



Quick Review

1. Find $\lim_{x \rightarrow -1} \frac{3x^2 - 2x + 1}{x^3 + 4}$

2. Let $f(x) = \text{int } x$. Find each limit.

(a) $\lim_{x \rightarrow -1} f(x)$

(b) $\lim_{x \rightarrow -1^+} f(x)$

(c) $\lim_{x \rightarrow -1} f(x)$

(d) $f(-1)$

3. Let $f(x) = \begin{cases} x^2 - 4x + 5, & x < 2 \\ 4 - x, & x \geq 2 \end{cases}$

Find each limit.

(a) $\lim_{x \rightarrow 2} f(x)$

(b) $\lim_{x \rightarrow 2^+} f(x)$

(c) $\lim_{x \rightarrow 2} f(x)$

(d) $f(2)$

Quick Review

In Exercises 4–6, find the remaining functions in the list of functions:

$f, g, f \circ g, g \circ f.$

$$4. \quad f(x) = \frac{2x-1}{x+5}, \quad g(x) = \frac{1}{x} + 1$$

$$5. \quad f(x) = x^2, \quad (g \circ f)(x) = \sin x^2, \quad \text{domain of } g = [0, \infty)$$

Quick Review

6. $g(x) = \sqrt{x-1}, \quad (g \circ f)(x) = \frac{1}{x}, \quad x > 0$

7. Use factoring to solve $2x^2 + 9x - 5 = 0$

8. Use graphing to solve $x^3 + 2x - 1 = 0$

Quick Review

In Exercises 9 and 10, let $f(x) = \begin{cases} 5 - x, & x \leq 3 \\ -x^2 + 6x - 8, & x > 3 \end{cases}$

9. Solve the equation $f(x) = 4$
10. Find a value of c for which the equation $f(x) = c$ has no solution.

Quick Review Solutions

1. Find $\lim_{x \rightarrow -1} \frac{3x^2 - 2x + 1}{x^3 + 4}$ 2

2. Let $f(x) = \text{int } x$. Find each limit.

(a) $\lim_{x \rightarrow -1^-} f(x)$ - 2 (b) $\lim_{x \rightarrow -1^+} f(x)$ - 1

(c) $\lim_{x \rightarrow -1} f(x)$ no limit (d) $f(-1)$ - 1

3. Let $f(x) = \begin{cases} x^2 - 4x + 5, & x < 2 \\ 4 - x, & x \geq 2 \end{cases}$

Find each limit.

(a) $\lim_{x \rightarrow 2^-} f(x)$ 1 (b) $\lim_{x \rightarrow 2^+} f(x)$ 2

(c) $\lim_{x \rightarrow 2} f(x)$ no limit (d) $f(2)$ 2

Quick Review Solutions

In Exercises 4–6, find the remaining functions in the list of functions:

f , g , $f \circ g$, $g \circ f$.

$$4. \quad f(x) = \frac{2x-1}{x+5}, \quad g(x) = \frac{1}{x} + 1$$

$$(f \circ g)(x) = \frac{x+2}{6x+1}, \quad x \neq 0$$

$$(g \circ f)(x) = \frac{3x+4}{2x-1}, \quad x \neq 5$$

$$5. \quad f(x) = x^2, \quad (g \circ f)(x) = \sin x^2, \quad \text{domain of } g = [0, \infty)$$

$$g(x) = \sin x, \quad x \geq 0$$

$$(f \circ g)(x) = \sin^2 x, \quad x \geq 0$$

Quick Review Solutions

6. $g(x) = \sqrt{x-1}, \quad (g \circ f)(x) = \frac{1}{x}, \quad x > 0$

$f(x) = \frac{1}{x^2} + 1, \quad x > 0 \quad (f \circ g)(x) = \frac{x}{x-1}, \quad x > 1$

7. Use factoring to solve $2x^2 + 9x - 5 = 0$ $x = \frac{1}{2}, -5$

8. Use graphing to solve $x^3 + 2x - 1 = 0$ $x \approx 0.453$

Quick Review Solutions

In Exercises 9 and 10, let $f(x) = \begin{cases} 5 - x, & x \leq 3 \\ -x^2 + 6x - 8, & x > 3 \end{cases}$

9. Solve the equation $f(x) = 4$ $x=1$

10. Find a value of c for which the equation $f(x) = c$
has no solution. $\text{Any } c \text{ in } [1, 2)$

What you'll learn about

- Definition of continuity at a point
- Types of discontinuities
- Sums, differences, products, quotients, and compositions of continuous functions
- Common continuous functions
- Continuity and the Intermediate Value Theorem

...and why

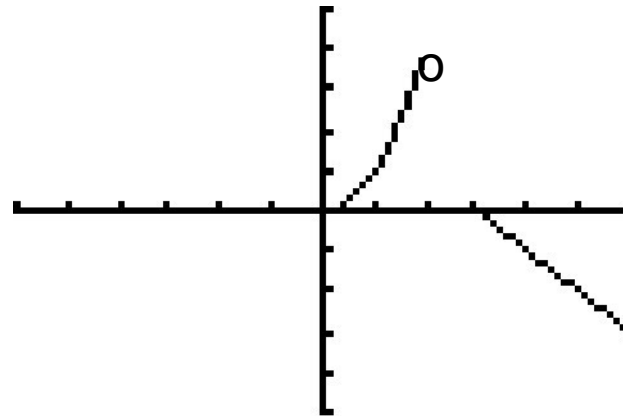
Continuous functions are used to describe how a body moves through space and how the speed of a chemical reaction changes with time.

Continuity at a Point

Any function $y = f(x)$ whose graph can be sketched in one continuous motion without lifting the pencil is an example of a continuous function.

Example Continuity at a Point

Find the points at which the given function is continuous and the points at which it is discontinuous.



Points at which f is continuous

At $x=0$

$$\lim_{x \rightarrow 0^+} f(x) = f(0)$$

At $x=6$

$$\lim_{x \rightarrow 6^-} f(x) = f(6)$$

At $0 < c < 6$ but not $2 \leq c < 3$

$$\lim_{x \rightarrow c} f(x) = f(c)$$

Points at which f is discontinuous

At $x=2$

$$\lim_{x \rightarrow 2} f(x) \text{ does not exist}$$

At $c < 0$, $2 < c < 3$, $c > 6$

these points are not in the domain of f

Continuity at a Point

Interior Point: A function $y = f(x)$ is continuous at an interior point c of its domain if $\lim_{x \rightarrow c} f(x) = f(c)$

Endpoint: A function $y = f(x)$ is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if $\lim_{x \rightarrow a^+} f(x) = f(a)$ or $\lim_{x \rightarrow b^-} f(x) = f(b)$ respectively.

Continuity at a Point

If a function f is **not continuous at a point c** , we say that f is **discontinuous at c** and c is a point of discontinuity of f .

Note that c need not be in the domain of f .

Continuity at a Point

The typical discontinuity types are:

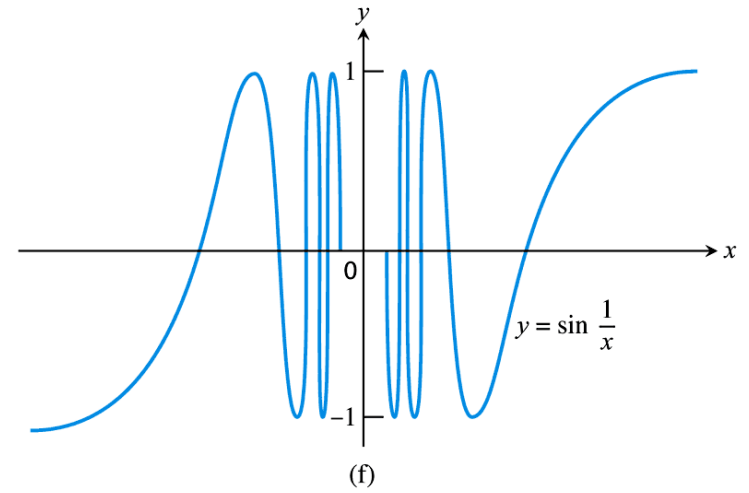
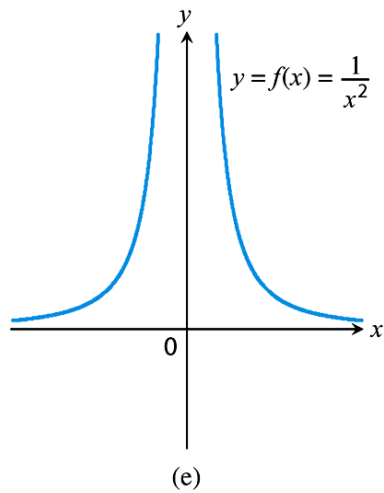
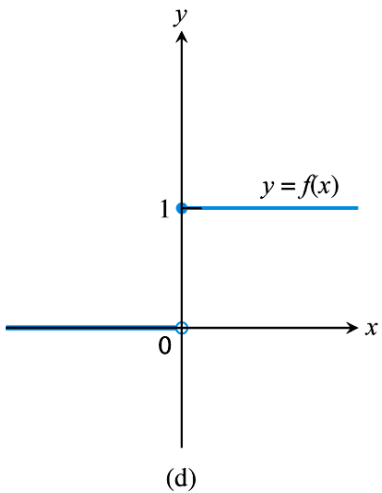
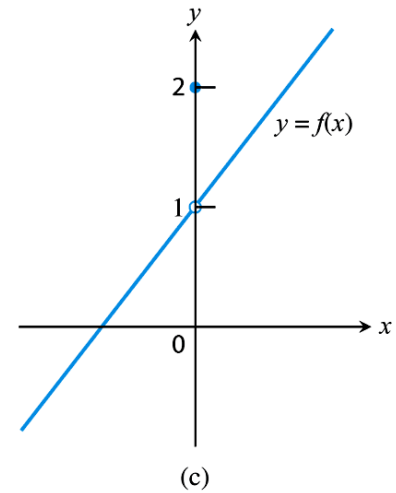
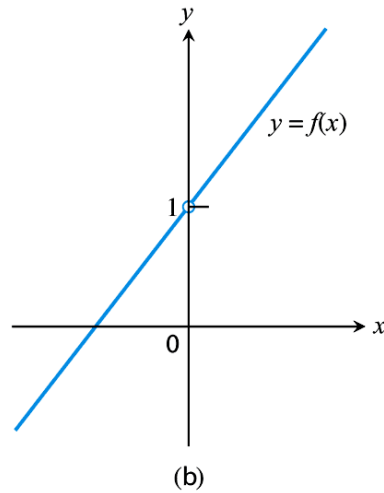
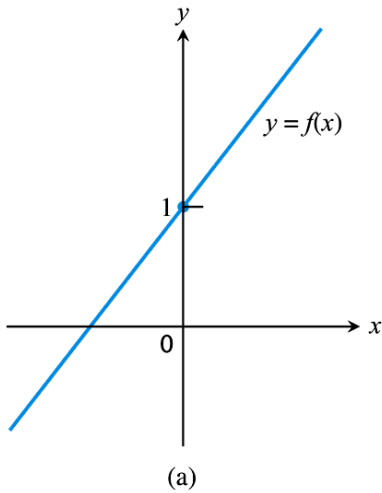
a) Removable (2.21b and 2.21c)

b) Jump (2.21d)

c) Infinite (2.21e)

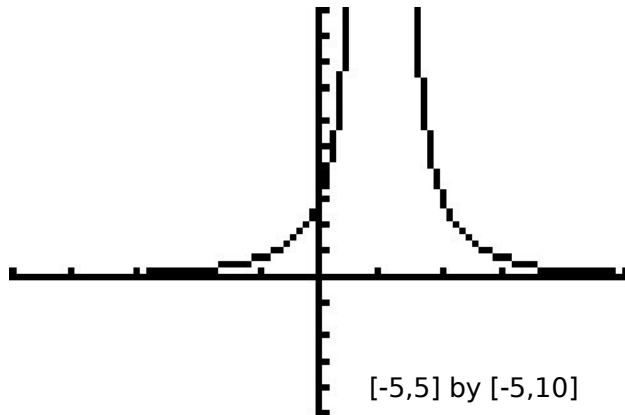
d) Oscillating (2.21f)

Continuity at a Point



Example Continuity at a Point

Find and identify the points of discontinuity of $y = \frac{3}{(x-1)^2}$



There is an infinite discontinuity at $x=1$.

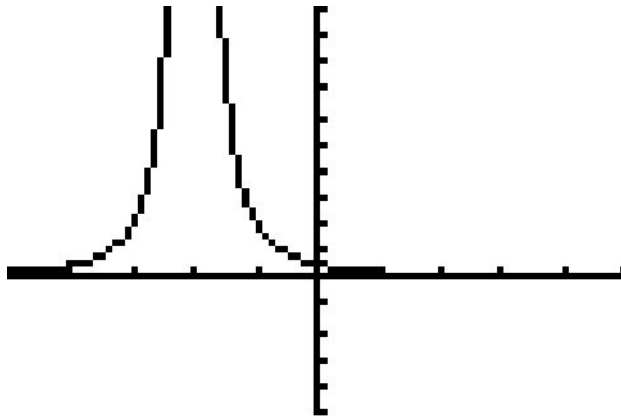
Continuous Functions

A function is **continuous on an interval** if and only if it is continuous at every point of the interval. A

continuous function is one that is continuous at every point of its domain. A continuous function need not be continuous on every interval.

Continuous Functions

The given function is a continuous function because it is continuous at every point of its domain. It does have a point of discontinuity at $x = -2$ because it is not defined there.



$$y = \frac{2}{(x+2)^2}$$

$[-5, 5]$ by $[-5, 10]$

Properties of Continuous Functions

If the functions f and g are continuous at $x=c$, then the following combinations are continuous at $x=c$.

1. Sums: $f + g$
2. Differences: $f - g$
3. Products: $f \cdot g$
4. Constant multiples: $k \cdot f$, for any number k
5. Quotients: $\frac{f}{g}$, provided $g(c) \neq 0$

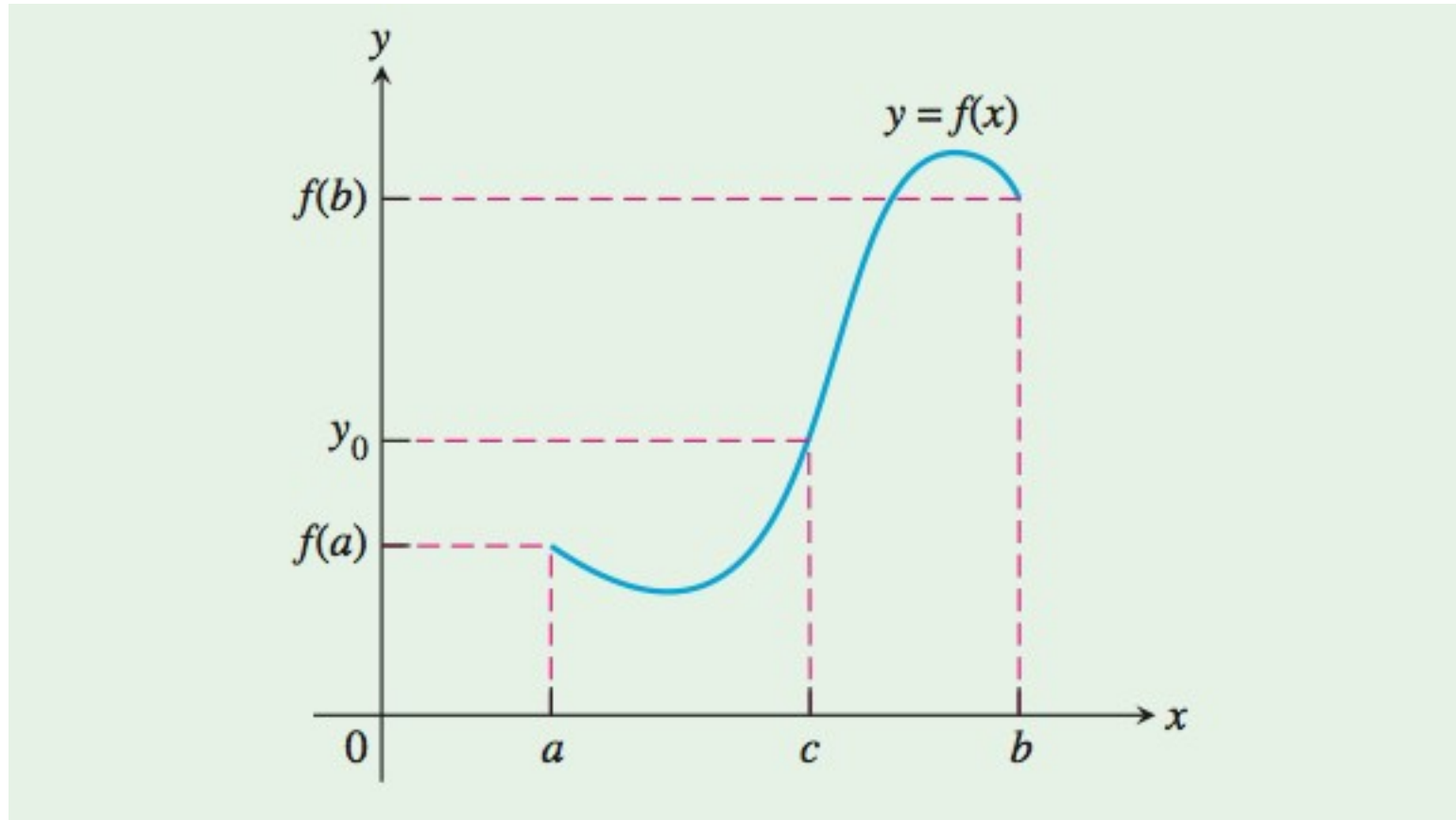
Composite of Continuous Functions

If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c .

Intermediate Value Theorem for Continuous Functions

A function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.

Intermediate Value Theorem for Continuous Functions



Intermediate Value Theorem for Continuous Functions

The Intermediate Value Theorem for Continuous Functions is the reason why the graph of a function continuous on an interval cannot have any breaks. The graph will be connected, a single, unbroken curve. It will not have jumps or separate branches.